Study of fluctuating fields behaviour in stochastic models of turbulence. QFT and RG approach.

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Outline

Stochastic dynamics, Quantum field theory, Renormalization group

- Stochastic dynamics defined by stochastic equation and correlator of a random force
- Stochastic model is equivalent to quantum field theory with an action functional
- Analysis of the stochastic model by renormalization group approach
- Interest: Kolmogorov regime, IR stability, UV fixed points, structure functions and constants of the model (Kolmogorov constant, Prandtl number etc.)

Helicity and Kolmogorov regime

Model of fully developed turbulence with helicity

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu_0 \Delta \mathbf{v} - \nabla P + \mathbf{f}^{\mathbf{v}} \tag{1}$$

Correlation function

$$\langle f_i(x) f_j(x') \rangle = \delta(t - t')(2\pi)^{-d} \int d\mathbf{k} R_{ij}(\mathbf{k}) D_0 k^{4-d-2\epsilon} \times \exp[i\mathbf{k}(\mathbf{x} - \mathbf{x}')] \tag{2}$$

Transverse projector, simulation of helicity

$$R_{ij}(\mathbf{k}) = P_{ij}(\mathbf{k}) + H_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j / k^2 + i\rho\varepsilon_{ijl} k_l / k \tag{3}$$

ε_{ijl} is Levi-Civita's antisymmetric tensor of rank 3
Physical condition for parameter of helicity

$$|\rho| \in [0, 1] \tag{4}$$

Helicity and Kolmogorov regime

Action functional of the model

$$S(\Phi) = \frac{1}{2}\int dx_1 dx_2 v'_i(x_1) D_{ij}(x_1;x_2) v'_j(x_2) + \int dx \mathbf{v}'[-\partial_v \mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{v} + \nu_0 \mathbf{v}] \quad (5)$$

Propagators

$$\langle v_i v'_j\rangle_0 = \langle v'_i v_j\rangle_0^* = \frac{P_{ij}(\mathbf{k})}{-i\omega + \nu_0 k^2} \quad (6)$$

$$\langle v_i v_j\rangle_0 = \frac{g_0 \nu_0^3 k^{4-d-2\epsilon} R_{ij}(\mathbf{k})}{(-i\omega + \nu_0 k^2)(i\omega + \nu_0 k^2)} \quad (7)$$

Interaction vertex

$$V_{ijl}(\mathbf{k}) = i(k_j \delta_{il} + k_l \delta_{ij}) \quad (8)$$

Helicity and Kolmogorov regime

Need to calculate 1 feynman diagram of one-loop approximation and 8 two-loop diagrams of the two-loop approximation

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RG functions (β, γ) lead to establishing IR stability of the model

...

Final result

$$\Omega = 2\epsilon\big[1 - \frac{3200\pi^4}{3}(-0.00825 + 0.00557\rho^2)\epsilon\big] \qquad (9)$$

The two-loop helical contribution is of the same order as the nonhelical one and at the same time has the opposite sign, nevertheless it cannot disturb the Stability of Kolmogorov scaling regime

Prandtl number under the influence of anisotropy

Passive scalar edvection

$$\partial_t \phi + (\mathbf{v} \cdot \partial)\phi = u_0 \nu_0 \Delta \phi + f \tag{10}$$

Navier-Stokes equation

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = \nu_0 \Delta \mathbf{v} - \partial P + f^v \tag{11}$$

Correlation function

$$\langle f_i^v(x) f_j^v(x') \rangle = \delta(t - t')(2\pi)^{-d} \int d\mathbf{k} R_{ij}(\mathbf{k}) d_f(k) \times \exp[i\mathbf{k}(\mathbf{x} - \mathbf{x}')] \tag{12}$$

Tensor projector

$$R_{ij}(\mathbf{k}) = \left(1 + \alpha_1 \frac{(\mathbf{n} \cdot \mathbf{k})^2}{k^2}\right) P_{ij}(\mathbf{k}) + \alpha_2 P_{is}(\mathbf{k}) n_s n_t P_{tj}(\mathbf{k}) \tag{13}$$

Prandtl number under the influence of anisotropy

Physical condition

$$\alpha_1 > -1, \qquad \alpha_2 > -1 \tag{14}$$

Set of fields

$$\Phi = \{\Theta, \mathbf{v}, \Theta', \mathbf{v}'\} \tag{15}$$

Action functional

$$\begin{aligned} S(\Phi) &= \frac{1}{2}\int \cdots + \int dt d^d\mathbf{x}(\Theta'\{-\partial_t - \mathbf{v}\cdot\partial + \nu_0 u_0[\Delta + \tau_0(\mathbf{n}\cdot\partial)^2]\}\Theta \\ &+ \mathbf{v}'\{-\partial_t - \mathbf{v}\cdot\partial + \nu_0[\Delta + \chi_{10}(\mathbf{n}\cdot\partial)^2]\}\mathbf{v} \\ &+ \nu_0 \mathbf{n}\cdot\mathbf{v}'[\chi_{20}\Delta + \chi_{30}(\mathbf{n}\cdot\partial)^2]\mathbf{n}\cdot\mathbf{v}) \end{aligned} \tag{16}$$

Propagators, Vertex $\rightarrow$ Feynman diagrams $\rightarrow$ IR stability $\rightarrow$ Anisotropic Prandtl number

Prandtl number under the influence of anisotropy

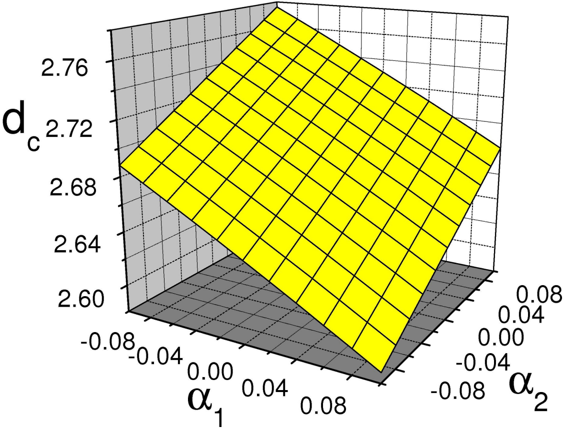

Figure: Dependence of the borderline dimension d_c on anisotropy parameters. Weak anisotropy limit.

Prandtl number under the influence of anisotropy

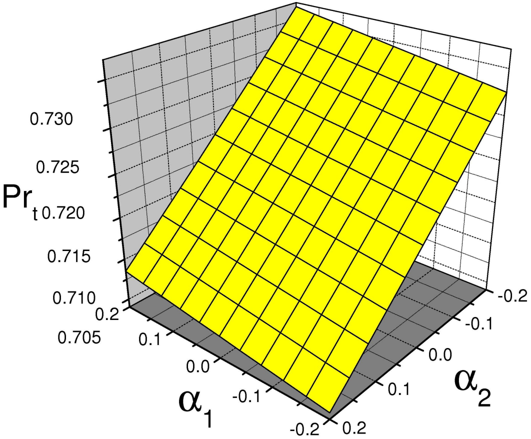

Figure: Dependence of the Prandtl number Pr_t on the anisotropy parameters. Weak anisotropy limit.

Prandtl number in two loops

Passive scalar advection

$$\partial_t \phi + (v_j \partial_j)\phi = \kappa_0 \Delta \phi + f \tag{17}$$

Navier-Stokes equation

$$\partial_t v_i + (v_j \partial_j) v_i = \nu_0 \Delta v_i - \partial_i P + f^v \tag{18}$$

Correlator of the random force

$$\langle f_i^v(t, \mathbf{x}) f_j^v(t', \mathbf{x}') \rangle = \delta(t - t')(2\pi)^{-d} \int d\mathbf{k} p_{ij}(\mathbf{k}) d_f(k) \times \exp[i\mathbf{k}(\mathbf{x} - \mathbf{x}')] \tag{19}$$

Prandtl number in two loops

Model with double set of fields

$$\Phi = \{\mathbf{v}, \phi, \mathbf{v}', \phi'\} \tag{20}$$

Action functional of the model

$$\begin{aligned} S(\Phi) &= \mathbf{v}' D_f \mathbf{v}'/2 + \mathbf{v}'[-\partial_t \mathbf{v} + \nu_0 \Delta \mathbf{v} - (\mathbf{v}\cdot\partial)\mathbf{v}] \\ &+ \phi'[-\partial_t \phi + \kappa_0 \Delta\phi - (\phi\partial)\phi + f] \end{aligned} \tag{21}$$

Action functional $\rightarrow$ Propagators, Vertex $\rightarrow$ Feynman diagrams $\rightarrow$ IR fixed points $\rightarrow$ RG functions (β, γ) $\rightarrow$ Turbulent Prandtl number

Prandtl number in two loops

Experimentally obtained values

$$Pr_t \in \langle 0.7, 0.9 \rangle \tag{22}$$

Two-loop value of the turbulent Prandtl number for $d = 3$

$$Pr_t = Pr_t^{(1)} + Pr_t^{(2)} = 0.7179 - 0.0128 = 0.7051 \tag{23}$$

Two-loop contribution to Pr_t is "only" about 2% and it has opposite sign with respect to the one-loop approximation.

Conclusion

- Analysis of an influence of helicity on the Kolmogorov regime in fully developed turbulence
- Analysis of an influence of anisotropy on anomalous scaling of a passive scalar field advected by the Navier-Stokes equation
- Calculation of the turbulent Prandtl number in two-loop approximation in a model of a passive scalar advection in fully developed turbulence

- List of publications
- List of visited conferences

Publications

- E. Jurčišinová, M. Jurčišin, R. Remecky and M. Scholtz, Physics of Particles and Nuclei Letters, Vol. 5, No. 3 (2008) 219-222
- E. Jurčišinová, M. Jurčišin, R. Remecky and M. Scholtz, International Journal of Modern Physics B, Vol. 22, No. 21 (2008) 3589-3617
- E. Jurčišinová, M. Jurčišin, R. Remecky, J. Phys. A: Math. Theor. 42 (2009) 275501
- E. Jurčišinová, M. Jurčišin, R. Remecky, Phys. Rev. E 79, 046319 (2009)
- E. Jurčišinová, M. Jurčišin, R. Remecky, Phys. Rev. E 80, 046302 (2009)
- J. Buša, E. A. Hayryan, E. Jurčišinová, M. Jurčišin, R. Remecky, Bulletin of PFUR, No 4. (2009)
- E. Jurčišinová, M. Jurčišin, R. Remecky, Comment on "Two-loop calculation of the turbulent Prandtl number" accepted, Phys. Rev. E

Presentations

- Renormalization Group 2008, Dubna, Russia
- PAMIR 2008, Presqu'île de Giens, France
- Small Triangle Meeting 2008, Medzilaborce, Slovakia
- EUROMECH, ETC12 2009, Marburg, Germany
- THMT'09, Rome, Italy
- Small Triangle Meeting 2009, Kysak, Slovakia